Constrained MPCI: A Weak Persistent Excitation Approach

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Model-predictive control and identification (MPCI) introduced by Genceli and Nikolaou in 1996 is a novel approach to the identification of processes under constrained model-predictive control. MPCI solves an on-line optimization problem that involves (a) a standard MPC objective; (b) standard MPC constraints; and (c) persistent excitation (PE) constraints. The on-line optimization problem is computationally demanding. To alleviate that problem, we take a frequency-domain approach to formulating the PE constraints. This approach relies on the following fact: a signal is persistently exciting of order n, if its two-sided power spectrum is nonzero at no fewer than n points. Therefore, persistently exciting input signals can be parametrized over a finite horizon as a sum of sinusoid terms with nonzero coefficients. Used in the MPCI framework, the last requirement generates a set of completely decoupled reverse-convex constraints that are combinatorially tractable from a computational point of view. The effectiveness of the proposed MPCI method is demonstrated through simulations. For the SISO systems studied, computation of the global optimum could be handled combinatorially in real-time using PC hardware.

Introduction

Identification of process dynamics involves excitation of the process with suitable input signals. This can be done in open-loop, where input–output data are acquired from the process without the influence of any closed-loop control action. However, we could have a number of situations when closed-loop identification of a process may be desirable or required. Situations when closed-loop identification may be desirable include:

- Change of operating conditions (e.g., distillation of crude oil, food processing)
- Process variation with time (e.g., due to fouling, catalyst deactivation, raw material variation)
- Presence of constraints during identification
- High cost of open-loop experimentation

The choice of process input signals is one of the fundamental issues in the design of an identification (open-loop or closed-loop) experiment (Mehra, 1974). A primary requirement for process inputs is that they be persistently exciting (Åström and Wittenmark, 1989; Goodwin and Sin, 1984). If the persistent excitation (PE) condition is satisfied, then convergence of the parameter estimation scheme can be guaranteed, under relatively mild assumptions, thereby permitting accurate identification. When closed-loop identification is coupled with controller adaptation, lack of PE may result in the bursting phenomenon, where the process outputs exhibit intermittent short periods of bursts, followed by periods of quiescence (Anderson, 1985). The PE condition also guarantees robustness of identification in the presence of noise and unmodeled dynamics (Sastry and Bodson, 1989). The precise mathematical formulation of conditions for PE can be found in several references such as Åström and Wittenmark (1989), Ljung (1987), Goodwin and Sin (1984), and Söderström and Stoica (1989).

While PE for process inputs is simple to guarantee in open-loop process-identification experiments, PE for process inputs in closed-loop identification experiments is more difficult to ensure. The main reason can be summarized as follows: the satisfaction of PE by process inputs depends on closed-loop properties that are uncertain because the closed-loop depends on the process, which needs to be identified in the first place. For example, to circumvent the problem that would be caused by the absence of PE, one may use

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the technique of dithering the process input or the setpoint to the closed-loop system. The dithering signals are usually random or sums of sinusoids at different frequencies. While this technique may achieve the objective of making the inputs persistently exciting, PE is achieved at the cost of compromising the control objective.

In other approaches to closed-loop identification, researchers have introduced the concept of dual control, where the conflicting objectives of identification and control are attempted to be met simultaneously. Fel'dbaum (1965) was the first to formulate the dual-control problem. He breaks the resulting control signal into what he calls a *testing* (probing) part and a *working* (controlling) part. Alster and Belanger (1974) proposed a suboptimal and dual-control scheme in which they considered a one-step-ahead control objective, subject to a constraint imposed on the information matrix. It should be stressed that the constraint considered by Alster and Belanger (1974) requires nonnegativity of the trace of the information matrix, which by itself does not guarantee invertibility of that matrix. Consequently, PE cannot be guaranteed by that inequality. Fu and Sastry (1992) perform a frequency-domain synthesis of open-loop optimal inputs for on-line identification and adaptive control. More recently, Genceli and Nikolaou (1996) approached the problem of closed-loop identification starting from a constrained model predictive control (MPC) framework. Their proposed approach, termed *model-predictive control and identification* (MPCI), establishes a new class of controllers that rely on the following on-line optimization problem: at each time step minimize a standard (e.g., quadratic) objective function subject to: (a) standard MPC constraints and (b) PE constraints on process inputs.

MPCI renders the PE condition for process inputs almost trivial to guarantee, thus guaranteeing parameter convergence (and eventually closed-loop stability) under very mild assumptions. Genceli and Nikolaou (1996) solve the MPCI on-line optimization problem by solving a sequence of semidefinite programming problems. These problems result from successive linearizations of the nonconvex quadratic matrix inequality constraints (that enforce PE) to linear matrix inequality constraints. Shouche et al. (1997) extend this approach to DARX models, with substantial reduction in the number of parameters to be identified. Shouche and Nikolaou (1996) presented a branch-and-bound algorithm that improves the numerical optimization approach of Genceli and Nikolaou (1996). The main drawback of all these schemes is that they are computationally expensive.

This article proposes a new formulation of the simultaneous MPCI problem. This new formulation relies on expressing the PE constraint in the frequency rather than in the time domain. The input variables are expressed in terms of certain prescribed *basis functions*, which in this case are chosen to be sinusoids. Desired PE properties can be realized by (a) appropriate choice of the base functions; (b) placing lower bounds on the absolute values of the coefficients of the different basis functions.

Practical advantages of the proposed formulation are

- Improved computational efficiency
- Straightforward incorporation into existing MPC software packages [c.f. predictive functional control (PFC) framework proposed by Abu et al., 1991].

The resulting on-line optimization problem consists of

- Quadratic objective
- Linear constraints
- Decoupled reverse-convex constraints on the coefficients of the sinusoids.

For small systems the global solution can be obtained by extensively solving a number of QP problems.

The rest of this article is organized as follows. In the next section we give a background of the PE problem. Next to that we present the new proposed formulation of MPCI for single-input, single-output (SISO) systems. Subsequently, we demonstrate the effectiveness of the proposed methodology through example simulation studies. Finally, we draw conclusions and discuss the future research potential of this work.

Background

The background section is organized in two parts. First, we briefly present the standard constrained MPC formulation and notation, then we discuss the PE concept.

MPC formulation and notation

The controlled process is described by the stable (SISO) deterministic autoregressive with exogenous input (DARX) model:

$$y(k+i/k) = -\sum_{l=1}^{na} a_l y(k+i-l/k) + \sum_{s=1}^{nb} b_l u(k+i-l) + d(k+i/k) + e(k+i/k), \quad (1)$$

where k is the current time; $y(k+i-l/k)$ are output predictions if $i-l>0$, and measured past values otherwise; $d(k+i/k)$ is the mean disturbance prediction estimated at time k as

$$d(k+i/k) = d(k/k) = y(k) - y(k/k-1); \quad (2)$$

and $e(k+i/k)$ is stationary random noise with zero mean.

The MPC on-line optimization problem is as follows:

$$\min_{\Delta u's} \left\{ w \sum_{l=1}^{p} (y(k+l/k) - y^{sp}(k+l/k))^2 + r \sum_{l=1}^{m} \Delta u(k+l-1)^2 \right\}$$

subject to

$$u_{\min} \le u(k+l) \le u_{\max} \quad (3)$$

$$\Delta u(k+l)\Delta u(k+l) - u(k+l-1)$$

$$|\Delta u(k+l)| \le \Delta u_{\max} \quad \text{for all } l = 1, \ldots, m-1$$

$$u(k+m+l) = u(k+m-1) \quad \text{for all } l = 0, \ldots, p-1$$

Define the following vectors:

$$y_f \equiv \begin{bmatrix} y(k+1/k) \\ y(k+2/k) \\ \vdots \\ y(k+p/k) \end{bmatrix}; \quad u_f \equiv \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+m-1) \end{bmatrix}$$

$$y_p \equiv \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y_1(k-na) \end{bmatrix}; \quad u_p \equiv \begin{bmatrix} u(k-1) \\ u(k-2) \\ \vdots \\ u(k-nb+1) \end{bmatrix}.$$

With the preceding vectors we can write all future output values in y_f in terms of future inputs u_f and all past outputs y_p, and inputs u_p as

$$y_f = B_f u_f + A_p y_p + B_p u_p. \tag{4}$$

The entries of matrices A_p, B_f, B_p are written in terms of the model coefficients. Using Eq. 4, we can cast the constrained MPC on-line problem as the following QP problem:

$$\min_{u_f} \frac{1}{2} u_f H u_f + g_T u_f$$

$$\text{subject to} \quad A u_f \leq b \tag{5}$$

The matrices H, A, and vectors g, b are in terms of the matrices A_p, B_f, B_p, and the current and past information contained in vectors y_p and u_p. In MPC the QP given by Eq. 5 is solved on-line at each instant k, and only the first element of the solution, $u(k)$, is implemented as the control input to the process. The whole procedure is repeated at the next instant $k+1$.

Identification and persistent excitation

In the identification problem we have input–output data sequences $\{u(k)\}$ and $\{y(k)\}$, and we seek to find the *best* model to fit the data. If we consider the DARX model structure:

$$y(k) = -\sum_{i=1}^{na} a_i y(k-i) + \sum_{i=1}^{nb} b_i u(k-i) + d(k) + e(k), \tag{6}$$

then we can write the previous equations as follows:

$$y(k) = \theta^T \varphi(k) + e(k), \qquad k = 1, \ldots, p \tag{7}$$

where

$$\theta = [a_1 \;\; a_2 \;\; \cdots \;\; a_{na} \;\; b_1 \;\; b_2 \;\; \cdots \;\; b_{nb} d]^T \tag{8}$$

$$\varphi = [-y(k-1) \;\; -y(k-2) \;\; \cdots \;\; -y(k-na)$$
$$u(k-1) \;\; u(k-2) \;\; \cdots \;\; u(k-nb) \;\; 1]^T. \tag{9}$$

We can then rewrite Eqs. 7 as

$$Y = \Phi^T \theta + e \tag{10}$$

where Y is the vector of output measurements, Φ is the matrix containing past outputs and inputs, θ is the parameter vector to be identified (Eq. 8), and e is the error vector. The matrix equation (Eq. 10) is solved in a least-squares sense. A number of algorithms, both batch and recursive, are readily available to solve such a problem. In all the variants of the least-squares-type solutions, we need to invert the matrix $\Phi\Phi^T$. The accuracy of the parameter estimate $(\hat{\theta})$, quantified by its covariance matrix estimate, is

$$E\left[(\theta - \hat{\theta})(\theta - \hat{\theta})^T\right] = (\Phi\Phi^T)^{-1} \sigma_e^2 \tag{11}$$

where σ_e^2 is the variance of the noise $e(k)$.

This brings us to the idea of PE. Qualitatively, the concept of PE consists of the following two requirements:

- Since the matrix $\Phi\Phi^T$ must be inverted, we require it to be nonsingular.
- For a desired level of accuracy, the matrix $(\Phi\Phi^T)^{-1}$ should be *small enough*, since the covariance of the parameter estimation error is directly proportional to it (Eq. 11). Equivalently, $\Phi\Phi^T$ should be *large enough* for accurate identification.

The qualitative requirements previously posed can be quantified as follows: consider the recursive least squares (RLS) algorithm without exponential forgetting given below:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-2)\varphi(k-1)}{1 + \varphi(k-1)^2 P(k-2)\varphi(k-1)} \times [y(k) - \varphi(k-1)^T \hat{\theta}(k-1)] \tag{12}$$

$$P(k-1) = \left[P(k-2) - \frac{P(k-2)\varphi(k-1)\varphi(k-1)^T P(k-2)}{1 + \varphi(k-1)^T P(k-2)\varphi(k-1)} \right], \tag{13}$$

where $\hat{\theta}(k)$ is the least-squares estimate of θ at time k; $P(k)$ is the covariance matrix at time k; $\hat{\theta}(0)$ and $P(-1)$ are initialized at time $k=0$.

Parameter convergence using the recursive least-squares algorithm is guaranteed if the following condition is satisfied (Goodwin and Sin, 1984):

$$\lim_{k \to \infty} \lambda_{\min}\left(\sum_{j=0}^{k-1} \varphi(j)\varphi(j)^T \right) = \infty. \tag{14}$$

This brings us to the mathematical definitions for persistence of excitation.

Definition A (Goodwin and Sin, 1984). A scalar input signal $\{u(t)\}$ is said to be *strongly persistently exciting* of order n if for all t there exists an integer l such that

$$\rho_1 I \geqslant \sum_{k=t}^{t+l} \begin{bmatrix} u(k+n) \\ u(k+n-1) \\ \vdots \\ u(k+1) \end{bmatrix} \times [u(k+n) \;\; u(k+n-1) \;\; \cdots \;\; u(k+1)] \geqslant \rho_2 I, \tag{15}$$

where $\rho_1 > \rho_2 > 0$.

Definition B (Goodwin and Sin, 1984). A scalar input signal $\{u(t)\}$ is said to be *weakly persistently exciting* of order n if

$$\rho_1 I \succcurlyeq \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\begin{bmatrix} u(t+n) \\ u(t+n-1) \\ \vdots \\ u(t+1)\end{bmatrix} \times [u(t+n) \quad u(t+n-1) \quad \cdots \quad u(t+1)] \succcurlyeq \rho_2 I, \quad (16)$$

where $\rho_1 > \rho_2 > 0$.

The second definition has an interpretation in the frequency domain:

Lemma A (Goodwin and Sin, 1984). A stationary input $\{u(t)\}$ is weakly persistently exciting of order n if its two-sided spectrum is nonzero at no fewer than n points.

Lemma B (Goodwin and Sin, 1984). For deterministic moving average (DMA) models, with n coefficients, the RLS algorithm converges, provided that the system input is weakly persistently exciting at least of order n; in particular, it suffices to use a stationary input whose spectral distribution is nonzero at n points or more.

Lemma C (Goodwin and Sin, 1984). For DARX models, the RLS algorithm converges provided that

(i) The system is stable

(ii) The input $\{u(t)\}$ has a spectral distribution function that is nonzero at $na + nb$ points or more

(iii) $A(z^{-1})$ and $B(z^{-1})$ are relatively prime, that is, they have no common roots, where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-1}, \quad (17)$$

$$B(z^{-1}) = 1 + b_1 z^{-1} + \cdots + b_{nb} z^{-1}. \quad (18)$$

If we have a signal consisting of n sinusoids with a nonzero bias, then the two-sided spectrum of the signal would consist of $2n+1$ nonzero points (Ljung, 1987). Thus, an order $2n+1$ weakly persistently exciting input u can be parametrized as follows:

$$u(t) = \alpha_0 + \sum_{l=1}^{n} \alpha_l \cos(\omega_l t),$$

$$\omega_i \neq \omega_j \quad \text{for} \quad i \neq j \quad \text{and} \quad \alpha_l \neq 0. \quad (19)$$

MPCI Formulation

With the background presented in the previous section, we can now formulate the proposed MPCI algorithm, which is the main contribution of this article.

At time k, we would like the input sequence, consisting of all past values, and current and future values (to be determined by the controller), to comprise certain frequencies, that is, we would like

$$u(k+i) \approx \sum_{l=1}^{m} \alpha_l \cos[\omega_l (k+i)], \quad \text{for} \quad -N \leq i \leq p-1, \quad (20)$$

where N is the number of past inputs that must satisfy the preceding condition. Writing Eq. 20 in vector–matrix form, we have at each time k

$$\begin{bmatrix} \mathbf{u}_{p,N} \\ \mathbf{u}_f \end{bmatrix} = \begin{bmatrix} \Gamma_p \\ \Gamma_f \end{bmatrix} \bar{\alpha}, \quad (21)$$

where $\mathbf{u}_f$ is as defined before;

$$\mathbf{u}_{p,N} \triangleq [u(k-N) \quad u(k-N+1) \quad \cdots \quad u(k-1)]^T \quad (22)$$

is the vector of past implemented inputs;

$$\bar{\alpha} \triangleq [\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_m]^T; \quad (23)$$

$$\Gamma_p \triangleq \begin{bmatrix} \cos[\omega_1(k-N)] & \cos[\omega_2(k-N)] & \cdots & \cos[\omega_m(k-N)] \\ \cos[\omega_1(k-N+1)] & \cos[\omega_2(k-N+1)] & \cdots & \cos[\omega_m(k-N+1)] \\ \vdots & \vdots & \ddots & \vdots \\ \cos[\omega_1(k-1)] & \cos[\omega_2(k-1)] & \cdots & \cos[\omega_m(k-1)] \end{bmatrix}; \quad (24)$$

$$\Gamma_f \triangleq \begin{bmatrix} \cos(\omega_1 k) & \cos(\omega_2 k) & \cdots & \cos(\omega_m k) \\ \cos[\omega_1(k+1)] & \cos[\omega_2(k+1)] & \cdots & \cos[\omega_m(k+1)] \\ \vdots & \vdots & \ddots & \vdots \\ \cos[\omega_1(k+m-1)] & \cos[\omega_2(k+m-1)] & \cdots & \cos[\omega_m(k+m-1)] \end{bmatrix}. \quad (25)$$

Using Eq. 21, we can find $\bar{\alpha}$ that best fits the input sequence $[\mathbf{u}_{p,N} \ \mathbf{u}_f]^T$ in a least-squares sense as follows:

$$\bar{\alpha} \approx \left[\Gamma_p^T \Gamma_p + \Gamma_f^T \Gamma_f\right]^{-1} \left[\Gamma_p^T \mathbf{u}_{p,N} + \Gamma_f^T \mathbf{u}_f\right]. \quad (26)$$

We can then write $\mathbf{u}_f$ as

$$\mathbf{u}_f = \mathbf{G}_\alpha \bar{\alpha} + \mathbf{h}_\alpha, \quad (27)$$

where

$$\mathbf{G}_\alpha = \Gamma_f^{-T} \Gamma_p^T \Gamma_p + \Gamma_f \quad (28)$$

$$\mathbf{h}_\alpha = -\Gamma_f^{-T} \Gamma_p^T \mathbf{u}_{p,N}. \quad (29)$$

We thus have a linear transformation between $\boldsymbol{u}_f$ and $\overline{\alpha}$ described by Eq. 27. Consequently, we can pose the QP problem in Eq. 5 in terms of the new decision variable, $\overline{\alpha}$ as follows:

$$\min_{\overline{\alpha}} \frac{1}{2}\overline{\alpha}^T \boldsymbol{H}_\alpha \overline{\alpha} + g_\alpha^T \overline{\alpha} \quad \text{subject to} \quad \boldsymbol{A}_\alpha \overline{\alpha} \le \boldsymbol{b}_\alpha, \tag{30}$$

where

$$\boldsymbol{H}_\alpha = \boldsymbol{G}_\alpha^T \boldsymbol{H} \boldsymbol{G}_\alpha \tag{31}$$

$$\boldsymbol{g}_\alpha = \boldsymbol{G}_\alpha^T [\boldsymbol{H}\boldsymbol{h}_\alpha + \boldsymbol{g}] \tag{32}$$

$$\boldsymbol{A}_\alpha = \boldsymbol{A}\boldsymbol{G}_\alpha \tag{33}$$

$$\boldsymbol{b}_\alpha = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{h}_\alpha . \tag{34}$$

Next, to impose the PE condition, we place constraints on the coefficients in $\overline{\alpha}$. These constraints force the selected frequency contents to be of some magnitude δ_i, which we call the level of excitation at frequency ω_i. This PE constraint is

$$|\alpha_i| \ge \delta_i > 0 \quad \text{for} \quad i = 2, \ldots, n_{\cos} + 1, \tag{35}$$

where $n_{\cos}$ is the required number of frequency components in the input for the desired order of PE.

Equations 30 to 35 give rise to a nonconvex QP problem with $2^{n_{\cos}}$ disjoint-convex feasible regions. This problem can be solved combinatorially to obtain the globally optimal solution for $\overline{\alpha}$ by solving a convex QP problem for each of the $2^{n_{\cos}}$ feasible regions. Once $\overline{\alpha}$ is obtained, we can get $\boldsymbol{u}_f$ from the linear transformation in Eq. 27. An algorithmic approach based on convex/reverse-convex global optimization is currently under investigation.

Remarks

1. We choose the first frequency $\omega_1 = 0$, so as to incorporate steady-state values for the inputs. Therefore, we require $|\alpha_i| \ge \delta_i > 0$, for $i = 2, \ldots, n_{\cos} + 1$

2. The frequencies $\{\omega_i\}_{i=1}^m$ are chosen so that $\max_i \omega_i$ satisfies the Nyquist sampling theorem, that is, $\max_i \omega_i \le (\pi/T_s)$, where T_s is the sampling period.

3. The frequency-spectrum condition on the input for PE is a very useful result, as pointed out by Ljung (1987). One can first prescribe the desired frequency spectrum. This may be realized by various input sequences. Thus, one could select a realization of the spectrum, taking practical aspects into consideration. The PE condition, however, does not specify the frequencies to be chosen, except that they satisfy the Nyquist sampling theorem. However, choosing input frequencies in the bandwidth of the system being identified would result in a higher information matrix, and thus parameter estimates with smaller covariance (Ljung, 1987).

4. We choose the number of sinusoids to be equal to the control horizon length m. A choice less than m would mean loss in the degrees of freedom as compared to the original problem formulation in Eq. 5. A choice greater than m would make the QP semidefinite (instead of definite) and solution of the nonconvex on-line optimization problem (Eqs. 30 to 35) would be more difficult.

5. To choose δ_i, we can first choose the desired signal-to-noise ratio (SNR). The minimum mean-squared value of the input sequence would be $\sum_{i=1}^{n_{\cos}}(\delta_i^2/2)$. Therefore we choose δ_i such that $\sum_{i=1}^{n_{\cos}}(\delta_i^2/2) \ge (\text{SNR}) \cdot \sigma_\epsilon^2/G$, where σ_ϵ^2 is the variance of the noise level in the output, and G is the steady-state gain of the system as estimated by an initial model of the system being identified.

6. In contrast to the standard MPC setup, where all inputs after time $k+m$ are equal to $u(k+m-1)$, that is,

$$u(k+m+l) = u(k+m-1) \quad \text{for all} \quad l = 0, 1, \ldots, p-m-1, \tag{36}$$

this formulation of MPCI requires all these inputs to be

$$u(k+m+l) = \sum_{j=1}^{m} \alpha_j \cos[\omega_j(k+l)] \quad \text{for all} \quad l = 0, 1, \ldots, p-m-1. \tag{37}$$

7. While starting up MPCI, we cannot use the PE constraint right away, because the inputs until that point in time most likely would not be PE, and the problem may be infeasible. To bypass this problem, we dither the input to the process by injecting a signal that contains the required frequencies for a short while before the MPCI algorithm actually takes effect.

Simulation Studies

Through the following simulations we wish to demonstrate the following:

- The effectiveness of the new proposed methodology in providing PE input signals for the purpose of identification of a system under closed-loop control. Comparisons are made to closed-loop identification with the addition of dithering signals.
- Comparison with other MPCI techniques proposed by Genceli and Nikolaou (1996) and Shouche et al. (1996). Comparisons are made between control performance as well as computational performance of the algorithm.

Example 1

Consider the following linear plant:

$$y(k) = 1.39y(k-1) - 0.49y(k-2) + 0.0564u(k-1) + 0.045u(k-2) + v(k), \tag{38}$$

where $v(k)$ is an output additive disturbance. This system settles to within $\pm 5\%$ of its steady state in 10 sampling intervals. The noise $v(k)$ is normally distributed with zero mean and a variance of 0.001. At time $k = 0$, the plant is modeled by

$$y(k) = 1.43y(k-1) - 0.513y(k-2) + 0.0844u(k-1) + 0.0408u(k-2) + d(k). \tag{39}$$

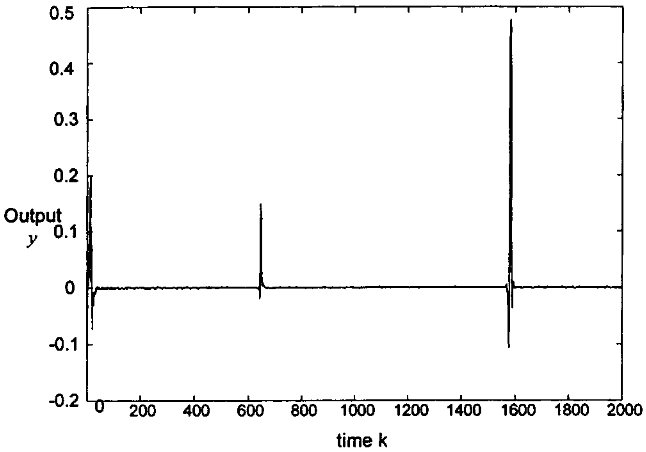

Figure 1. Bursting phenomenon observed in the absence of PE for Example 1.

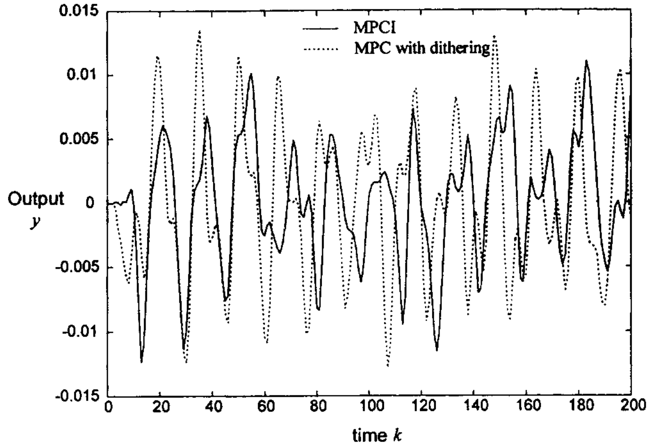

Figure 3. Comparison of outputs: MPCI and MPC with dithering for Example 1.

The objective of the controller is to regulate the system output $y(k)$. The following controllers are compared.

MPC with Process Model Parameter Adaptation. The controller is heuristically tuned as follows:

$$m = 4$$
$$p = 14$$
$$r = 0.5$$
$$w = 1.0.$$

The process model parameters are identified at each time instant using the RLS algorithm. The PE constraint is not imposed.

MPCI. Values of the tuning parameter m, p, r, w remain as before. In addition, the selected frequencies for PE (Eq. 20) are

$$\omega = (0 \quad 0.3913 \quad 0.8200 \quad 1.1251)^T \quad \text{radians per unit time}$$

We have

$$n_{\cos} = 2$$
$$\delta = 0.01,$$

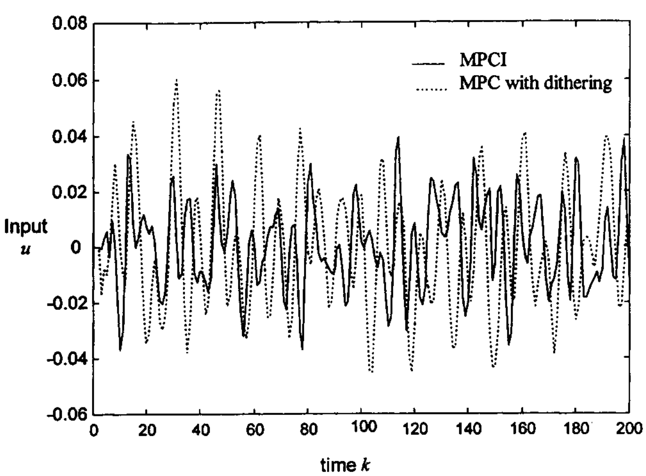

Figure 2. Comparison of inputs: MPCI and MPC with dithering for Example 1.

which means that we require that the magnitude components of frequencies ω_2 and ω_3 are at least 0.01, as described in the previous section. To solve this problem, we solve $2^2 = 4$ QP problems at each time instant k.

MPC with Sinusoidal Dithering to the Control Input. The external dithering signal that is added is obtained by the addition of two sinusoids, each with magnitude of 0.01 as follows:

$$u_{\text{dith}}(k) = \sum_{i=2}^{3} 0.01 \cos(\omega_i k) \tag{40}$$

with ω_2 and ω_3 being the same as in the MPCI design.

Simulation Results. The output response for MPC with process model adaptation, shown in Figure 1, demonstrates the well-known phenomenon of bursting in adaptive systems. Figure 2 shows the inputs, Figure 3 shows process outputs, and Figure 4 demonstrates the convergence of the four identified parameters with time.

Comparing the inputs and outputs plots for MPCI and MPC with process input dithering, we see that MPCI results in smaller variation in the process output. It is arguable here that external sinusoidal dithering results in higher variances for both the input and output, thereby providing the identification scheme with more information content. However, the objective of simultaneous closed-loop control and identification requires both the identification and control objectives to be met. With external dithering it turns out that we end up with more energy in the input and output sequences than asked for. Regarding parameter tracking, both schemes are comparable in terms of the quality of the estimates, although the parameters obtained using MPC with dithering have smaller covariance. However, it should be noted here that the improved parameter estimates produced by MPC with dithering do not really improve the control performance, relative to MPCI. The variances for the two output responses are as follows:

$$\text{MPCI:} \quad 2.27 \times 10^{-5}$$
$$\text{MPC with dithering:} \quad 3.56 \times 10^{-5}.$$

In the formulation of the problem we write the input se-

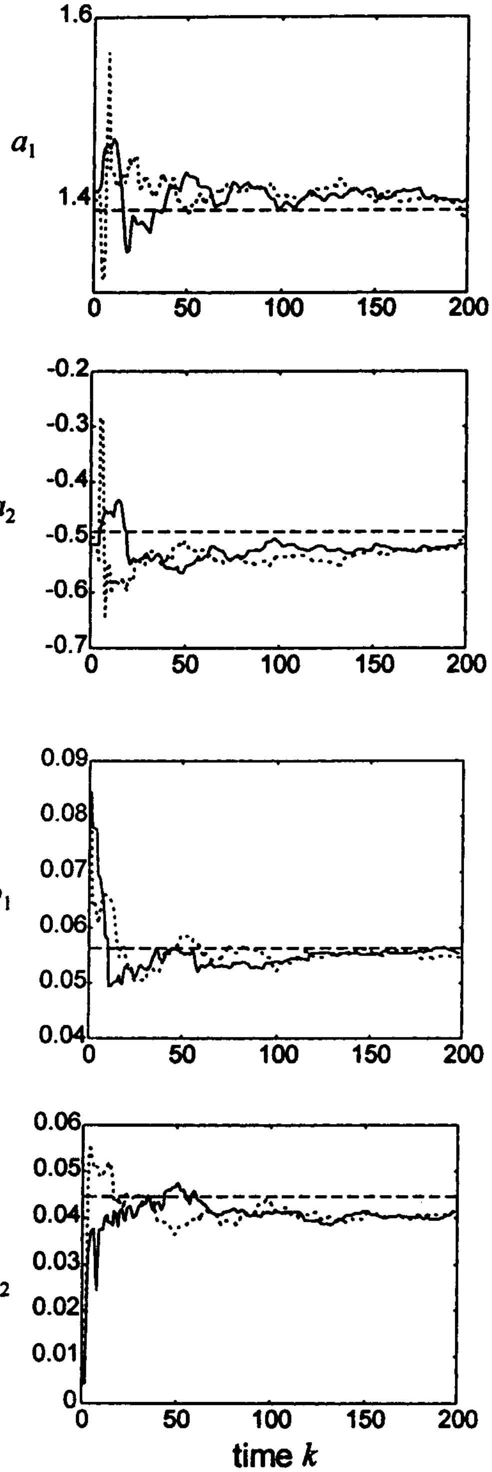

Figure 4. Parameter tracking: MPCI (——); MPC with dithering (· · · ·); and actual parameter values (- - -) for Example 1.

quence as a sum of sinusoids. We then find the least-squares best fit for the input sequence in terms of a chosen set of sinusoids. We need to check whether the resulting closed-loop input sequence does have the intended frequencies. To do so, MPCI is run for 2,000 time units and a spectral analysis of the input sequence resulting from the MPCI scheme reveals a power spectrum estimate, which is shown in Figure 5. It

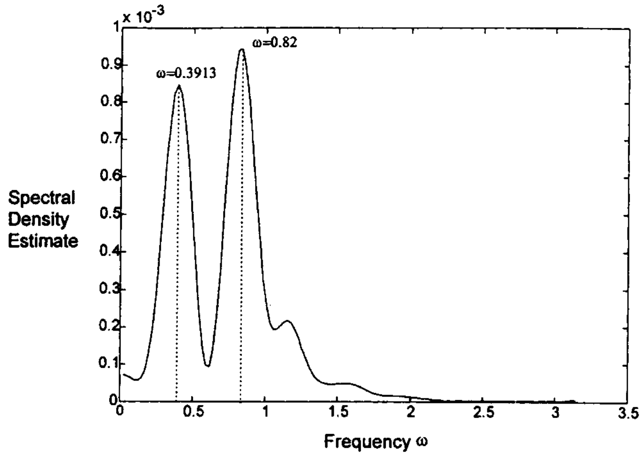

Figure 5. Power spectrum estimate of input from MPCI scheme for Example 1.

can be noticed from the spectral estimate that the input does indeed have the desired frequencies prescribed by the design.

Next we compare the performance of the new proposed MPCI scheme with the scheme proposed by Genceli and Nikolaou (1996) and Shouche et al. (1997). Given in Figures 6 and 7 are the input and output plots, respectively. As can be seen from the responses, both the input and the output of the earlier MPCI scheme have a higher variance. This can be for two reasons. The first is that in the earlier MPCI scheme of Genceli and Nikolaou (1996) and Shouche et al. (1997), the PE condition used is the strong PE condition defined in Definition A, whereas in our algorithm we use the weak PE condition. The strong PE condition, as its name suggests, is a stricter constraint than the weak PE condition. Another reason for a larger variability is that the strong PE constraint results in a nonconvex quadratic matrix inequality. The resulting optimization problem is solved using successive semidefinite programming (SDP), and the solution arrived at is not always the global solution.

Another big difference between the two techniques of MPCI lies in the computational performance. Table 1 shows the difference in CPU times for on-line execution of the re-

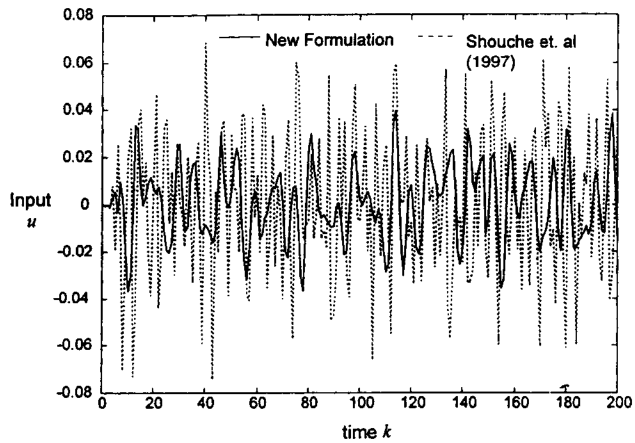

Figure 6. Comparison of inputs: MPCI (new formulation) and MPCI (Shouche et al., 1997) for Example 1.

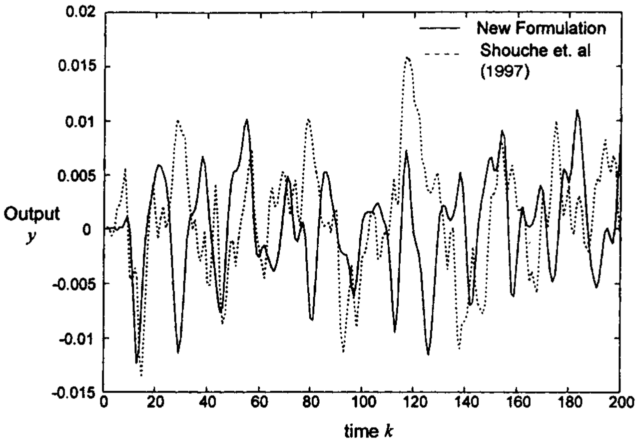

Figure 7. Comparison of outputs: MPCI (new formulation) and MPCI (Shouche et al., 1997) for Example 1.

spective optimization algorithms at each time k. The simulations were run on a 32-MB/90-MHz Pentium PC. While the respective codes were not optimized for speed of execution, the results are indicative of relative performance. The reason for the orders-of-magnitude difference is that in the original formulation of MPCI the strong PE condition is used, which results in quadratic matrix inequalities (QMIs). These QMIs can be bounded from below by linearizing them at some point, thus generating linear matrix inequality (LMI) constraints. The LMI-constrained problem is solved using SDP. The entire on-line optimization problem is solved through successive linearizations of the QMIs at the solutions of the LMI-constrained problems at previous iterations. This solution methodology is computationally slow.

Example 2

For this example we consider a CSTR in which the reaction $A \rightarrow B$ is taking place. The system is described by the following nonlinear system of first-order ODEs:

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Ai}(t) - C_A(t)) - k_0 e^{-(E/RT(t))} C_A(t) \quad (41)$$

$$\frac{dT}{dt} = \frac{F}{V}(T_i(t) - T(t)) + Jk_0 e^{-(E/RT(t))} C_A(t) - \frac{UA}{\rho C_p V}(T(t) - T_c(t)). \quad (42)$$

The numerical values of the various parameters are given in Table 2, and the steady-state values in Table 3.

We apply MPCI to the system just discussed, with C_A as the output variable to be controlled and the input control variable is the inlet temperature T_i. We transform all the variables centered around zero as follows:

Table 1. Comparison of Computational CPU Times

Algorithm	New MPCI Formulation	MPCI Using SDP (Shouche et al., 1997)
CPU time (in s)	~ 0.4 s	~ 60 s

Table 2. CSTR Parameter Values

F/V (h^{-1})	k_0 (h^{-1})	E/R (K)	J ($m^3 \cdot K/mol$)	$UA/\rho C_p$ (m^3/h)
0.8331	7.08×10^{-7}	8375	0.02775	2.8

$$x_n = \frac{x - x_s}{x_s},$$

where x_n is the new transformed variable; x is the old variable; and x_s is the steady-state value of the variable. This is done for each variable, using the steady-state values in Table 3.

Under steady state the system is initially assumed to be operating at the values shown in Table 3. Disturbances to the system are subjected through changes in the inlet feed concentration. The system is sampled at a rate of 6 min (0.1 h). The MPCI controller is tuned as follows:

$$m = 5$$
$$p = 15$$
$$r = 1.0$$
$$w = 1.0.$$

The linear DARX model used initially by the MPCI controller is as follows:

$$y(k) = 1.0381y(k-1) - 0.2646y(k-2) - 0.8113u(k-1) + d(k). \quad (43)$$

The input and input move constraints are as follows:

$$-1 \leq u \leq 1$$
$$|\Delta u| \leq 0.5.$$

We choose for the MPCI scheme the following frequencies

$$\omega = (0 \quad 0.3245 \quad 0.6808 \quad 1.0581 \quad 1.2230)^T \quad \text{rad/unit sampling time.}$$

The frequencies are chosen spaced between 0 and $\pi/2$. We choose

$$n_{pe} = 2$$
$$\delta = 0.05.$$

For the purpose of comparison we examine the performance of MPC with the following dithering signal:

$$u_{\text{dith}}(k) = 0.05 \sum_{i=2}^{3} \cos(\omega_i k). \quad (44)$$

Table 3. CSTR Steady-State Values

C_{Ais} (mol/m^3)	C_{As} (mol/m^3)	T_{is} (K)	T_{cs} (K)	T_s (K)
8008	393.2	373.3	532.6	547.6

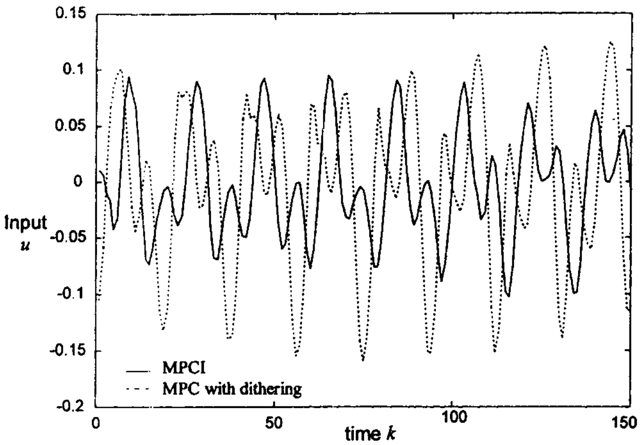

Figure 8. Comparison of inputs: MPCI and MPC with dithering for Example 2.

The closed-loop input and output responses corresponding to MPCI and MPC with process input dithering are given in Figures 8 and 9, respectively. Again, as can be observed from the responses, MPCI gives a smaller deviation in the input and output as compared to MPC with sinusoidal dithering. The output variances for the two systems are

$$\text{MPCI: } 0.0192$$
$$\text{MPC with dithering: } 0.0390.$$

The model identified using MPCI is

$$y(k) = 1.239y(k-1) - 0.4192y(k-2) - 0.7330u(k-1) + d(k), \quad (45)$$

and the model identified using MPC with dithering is

$$y(k) = 1.2798y(k-1) - 0.4634y(k-2) - 0.7270u(k-1) + d(k). \quad (46)$$

To compare the two models obtained we run MPC using each of these models, as well as the originally used model given by Eq. 43. The plant is to be controlled at the operating

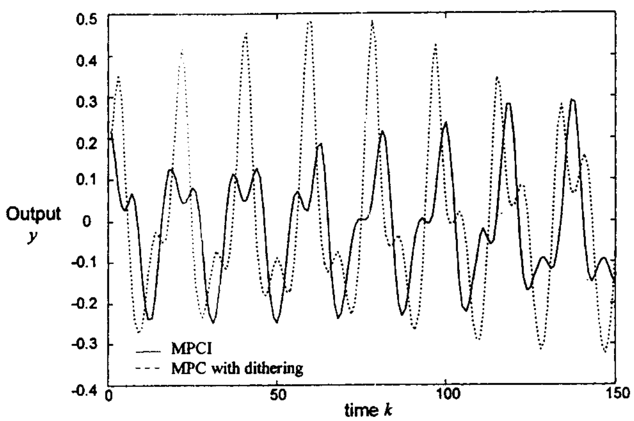

Figure 9. Comparison of outputs: MPCI and MPC with dithering for Example 2.

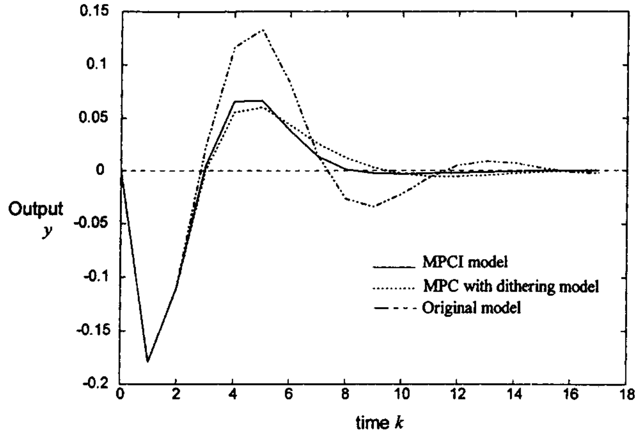

Figure 10. Disturbance rejection using the different models.

point at which the preceding identifications were made. Responses to a disturbance in the inlet feed concentration are given in Figure 10. As can be observed from Figure 10, both identification schemes yield models that result in better performance than the original model. It can also be seen that the model from MPCI yields a slightly better controller than the one using the model from MPC with the dithering method. This can be accounted for in terms of the input energy that is put into the system. Since the input to the nonlinear plant in the dithering case has a greater variation about the operating point, the resulting model has greater error introduced into it due to the nonlinearity. Thus we see that MPCI, by providing the desired amount of energy, arrives at a better model than the dithering scheme.

Conclusions

In this article, a new formulation is given for simultaneous MPC and identification of SISO systems, based on the weak PE condition in the frequency domain. As demonstrated, this new formulation has a number of advantages. The resulting optimization problem has a number of simple reverse-convex constraints, and may be solved combinatorially for small systems using standard efficient QP techniques. This aspect of the proposed scheme makes it attractive from the viewpoint of computational efficiency, in comparison to MPCI schemes proposed earlier.

The proposed scheme compares well with MPC with dithering. It results in the input signal having just the desired amount of energy as specified by the design, and thereby minimally perturbing the control objective.

In future work, this algorithm will be extended to multivariable systems.

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